# Mathematics

Single correct answer type:

81. $\sin^{-1}(\sin 5) > x^2 - 4x$ holds if

(A) $x = 2 - \sqrt{9 - 2\pi}$      (B) $x = 2 + \sqrt{9 - 2\pi}$

(C) $x > 2 + \sqrt{9 - 2\pi}$      (D) $x \in \left(2 - \sqrt{9 - 2\pi}, 2 + \sqrt{9 - 2\pi}\right)$

Solution: (D)

82. A value of c for which conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval $[1, 3]$ is

(A) $\log_3 e$      (B) $\log_e 3$      (C) $2 \log_3 e$      (D) $\frac{1}{2} \log_3 e$

Solution: (C)

83. Negation of the proposition: If we control population growth, we prosper

(A) If we do not control population growth, we prosper

(B) If we control population growth, we do not prosper

(C) We control population but we do not prosper

(D) We do not control population, but we prosper

Solution: (C)

84. The equation $z\bar{z} + (2 - 3i)z + (2 + 3i)\bar{z} + 4 = 0$ represents a circle of radius

(A) 2      (B) 3      (C) 4      (D) 6

Solution: (B)

85. The function $f(x) = \sin x - kx - c$, where $k$ and $c$ are constants, decreases always when

(A) $k > 1$      (B) $k \geq 1$      (C) $k < 1$      (D) $k \leq 1$

Solution: (B)

86. Equation $\frac{1}{r} = \frac{1}{8} + \frac{3}{8}\cos\theta$ represents

(A) A rectangular hyperbola

(B) A hyperbola

(C) An ellipse

(D) A parabola

Solution: (B)

87. The acceleration of a sphere falling through a liquid is $(30 - 3v)$ $cm/s^2$ where v is its speed in $cm/s$. The maximum possible velocity of the sphere and the time when it is achieved are

(A) $10$ $cm/s$ after 10 second

(B) $10$ $cm/s$ instantly

(C) $10$ $cm/s$, will never be achieved

(D) $30$ $cm/s$, after 30 second

Solution: (C)

88. A straight line parallel to the line $2x - y + 5 = 0$ is also a tangent to the curve $y^2 = 4x + 5$. Then the point of contact is

(A) (2, 1)         (B) (-1, 1)         (C) (1, 3)         (D) (3, 4)

Solution: (B)

89. Value of $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ is

(A) $\frac{\pi}{2}$     (B) $\frac{-\pi}{2}$     (C) $\frac{\pi}{4}$     (D) None of these

Solution: (C)

90. The range of the function $f(x) = \frac{1}{2 - \cos 3x}$ is

(A) $(-2, \infty)$         (B) $[-2, 3]$         (C) $\left[\frac{1}{3}, 1\right]$         (D) $\left(\frac{1}{2}, 1\right)$

Solution: (C)

91. The area bounded by $y = 1 = |x|, y = 0$ and $|x| = \frac{1}{2}$ will be:

(A) $\frac{3}{4}$        (B) $\frac{3}{2}$        (C) $\frac{5}{4}$        (D) None of these

Solution: (C)

92. The value of $x$ obtained from the equation $\begin{vmatrix} x+\alpha & \beta & \gamma \\ \gamma & x+\beta & \alpha \\ \alpha & \beta & x+\gamma \end{vmatrix} = 0$ will be

(A) 0 and $-(\alpha + \beta + \gamma)$        (B) 0 and $\alpha + \beta + \gamma$

(C) 1 and $(\alpha - \beta - \gamma)$        (D) 0 and $\alpha^2 + \beta^2 + \gamma^2$

Solution: (A)

93. The solution of the differential equation $\log x \frac{dy}{dx} + \frac{y}{x} = \sin 2x$ is

(A) $y \log|x| = C - \frac{1}{2} \cos x$        (B) $y \log|x| = C + \frac{1}{2} \cos 2x$

(C) $y \log|x| = C - \frac{1}{2} \cos 2x$        (D) $xy \log|x| = C - \frac{1}{2} \cos 2x$

Solution: (C)

94. $\lim\limits_{x \to \infty} \left( \frac{x^2}{3x-2} - \frac{x}{3} \right) =$

(A) $\frac{1}{3}$        (B) $\frac{2}{3}$        (C) $\frac{-2}{3}$        (D) $\frac{2}{9}$

Solution: (D)

95. If $\left( (\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) \right) . (\vec{a} \times \vec{d}) = 0$, then which of the following is always true?

(A) $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are necessarily coplanar

(B) either $\vec{a}$ or $\vec{d}$ must lie in the plane of $\vec{b}$ and $\vec{c}$

(C) Either $\vec{b}$ or $\vec{c}$ must lie in the plane of $\vec{a}$ and $\vec{d}$

(D) Either $\vec{a}$ or $\vec{b}$ must lie in the plane of $\vec{c}$ and $\vec{d}$

Solution: (C)

96. Let $A$ be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$, and $B(1,7)$ and $D(4-2)$ are points on the circle then, if tangents be drawn at B and D, which meet at C, then area of quadrilateral ABCD is-

(A) 150       (B) 75       (C) 75/2       (D) None of these

Solution: (B)

97. $\int_0^1 [f(x)g''(x) - f''(x)g(x)]\, dx$ is equal to:

[Given $f(0) = g(0) = 0$]

(A) $f(1)\, g(1) - f(1)g'(1)$       (B) $f(1)g'(1) + f'(1)g(1)$

(C) $f(1)g'(1) + f'(1)g(1)$       (D) $f(1)g'(1) - f'(1)g(1)$

Solution: (C)

98. If $z = \dfrac{7-i}{3-4i}$ then $z^{14} =$

(A) $2^7$       (B) $2^7 i$       (C) $2^{14} i$       (D) $-2^7 i$

Solution: (D)

99. The difference between greatest and least value of $f(x) = 2\sin x + \sin 2x$, $x \in \left[0, \dfrac{3\pi}{2}\right]$ is-

(A) $\dfrac{3\sqrt{3}}{2}$       (B) $\dfrac{3\sqrt{3}}{2} - 2$       (C) $\dfrac{3\sqrt{3}}{2} + 2$       (D) None of these

Solution: (C)

100. A and B are two independent witnesses (i.e., there is no collision between them) in a case. The probability that A will speak the truth is $x$ and the probability that B will speak the truth is $y$. A and B agree in a certain statement. The probability that the statement is true is

(A) $\dfrac{x-y}{x+y}$       (B) $\dfrac{xy}{1+x+y+xy}$

(C) $\frac{x-y}{1-x-y+2xy}$ (D) $\frac{xy}{1-x-y+2xy}$

Solution: (D)

101. A and B are events such that $P(A \cup B) = \frac{3}{4}, P(A \cap B) = \frac{1}{4}, P(\bar{A}) = \frac{2}{3}$ then $(\bar{A} \cap B)$ is

(A) $\frac{5}{12}$ (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{1}{4}$

Solution: (A)

102. The line which passes through the origin ad intersect the two lines
$\frac{x-1}{2} = \frac{y+3}{4} = \frac{z-5}{3}, \frac{x-4}{2} = \frac{y+3}{3} = \frac{z-14}{4}$, is

(A) $\frac{x}{1} = \frac{y}{-3} = \frac{z}{5}$ (B) $\frac{x}{-1} = \frac{y}{3} = \frac{z}{5}$

(C) $\frac{x}{1} = \frac{y}{3} = \frac{z}{-5}$ (D) $\frac{x}{1} = \frac{y}{4} = \frac{z}{-5}$

Solution: (A)

103. If $u_n = \int_0^{\frac{\pi}{4}} \tan^n \theta \, d\theta$ then $u_n + u_{n-2}$ is:

(A) $\frac{1}{n-1}$ (B) $\frac{1}{n+1}$ (C) $\frac{1}{2n-1}$ (D) $\frac{1}{2n+1}$

Solution: (A)

104. Ten different letters of an alphabet are given, words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated is

(A) 69760 (B) 30240 (C) 99784 (D) None of these

Solution: (A)

105. The area bounded by $f(x) = x^2, 0 \leq x \leq 1, g(x) = -x + 2, 1 \leq x \leq 2$ and $x$-axis is

(A) $\frac{3}{2}$ (B) $\frac{4}{3}$ (C) $\frac{8}{3}$ (D) None of these

Solution: (D)

106. The condition that the line $\frac{x}{p} + \frac{y}{q} = 1$ be a normal to the parabola $y^2 = 4ax$ is

(A) $p^3 = 2ap^2 + aq^2$  (B) $p^3 = 2aq^2 + ap^2$

(C) $q^3 = 2ap^2 + aq^2$  (D) None of these

Solution: (A)

107. A random variable $X$ has the probability distribution

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| p(X) | 0.15 | 0.23 | 0.12 | 0.10 | 0.20 | 0.08 | 0.07 | 0.05 |

For the events $E = \{X$ is a prime number$\}$ and $F = \{X < 4\}$, then $P(E \cup F)$ is

(A) 0.50  (B) 0.77  (C) 0.35  (D) 0.87

Solution: (B)

108. The value of $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{7}{8}$ is

(A) $\tan^{-1}\frac{7}{8}$  (B) $\cot^{-1} 15$  (C) $\tan^{-1} 15$  (D) $\tan^{-1}\frac{15}{24}$

Solution: (C)

109. The parabola having its focus at (3, 2) and directrix along the y-axis has its vertex at

(A) $(2, 2)$  (B) $\left(\frac{3}{2}, 2\right)$  (C) $\left(\frac{1}{2}, 2\right)$  (D) $\left(\frac{2}{3}, 2\right)$

Solution: (B)

110. The rank of the matrix $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix}$ is

(A) 1 if $a = 6$  (B) 2 if $a = 1$

(C) 3 if $a = 2$  (D) 1 if $a = 4$

Solution: (B)

111. If $f(x) = \begin{vmatrix} \cos x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$, then $\int_0^{\frac{\pi}{2}} f(x)\, dx$ is equal to

(A) $\frac{1}{4}$      (B) $-\frac{1}{3}$      (C) $\frac{1}{2}$      (D) 1

Solution: (B)


112. The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z-1}{-6}$ is

(A) 1      (B) 2      (C) 4      (D) $2\sqrt{3}$

Solution: (A)


113. The tangent lines to the curve $y^2 = 4ax$ at points where $x = a$, are

(A) Parallel      (B) Perpendicular

(C) Inclined at $60^o$      (D) Inclined at $30^o$

Solution: (B)


114. If the eccentricity of the hyperbola $x^2 - y^2 cosec^2\alpha = 25$ is $\sqrt{5}$ times the eccentricity of the ellipse $x^2 cosec^2\alpha + y^2 = 5$, then $\alpha$ is equal to:

(A) $\tan^- \sqrt{2}$      (B) $\sin^{-1}\sqrt{\frac{3}{4}}$      (C) $\tan^{-1}\sqrt{\frac{2}{5}}$      (D) $\sin^{-1}\sqrt{\frac{2}{5}}$

Solution: (A)


115. The conditional $(p \wedge q) \Rightarrow p$ is

(A) A tautology

(B) A fallacy i.e., contradiction

(C) Neither tautology nor fallacy

(D) None of these

Solution: (A)

116. The set of points of discontinuity of the function $f(x) = \lim\limits_{n \to \infty} \dfrac{(2 \sin x)^{2n}}{3^n - (2 \cos x)^{2n}}$ is given by

(A) $R$

(B) $\left\{ n\pi \pm \dfrac{\pi}{3}, n \in I \right\}$

(C) $\left\{ n\pi \pm \dfrac{\pi}{6}, n \in I \right\}$

(D) None of these

Solution: (C)


117. The volume V and depth $x$ of water in vessel are connected by the relation $V = 5x - \dfrac{x^2}{6}$ and the volume of water is increasing, at the rate of $5\,cm^3/sec$, when $x = 2cm$. The rate at which the depth of water is increasing, is

(A) $\dfrac{5}{18}\,cm/sec$ 
(B) $\dfrac{1}{4}\,cm/sec$ 
(C) $\dfrac{5}{16}\,cm/sec$ 
(D) None of these

Solution: (D)


118. If vectors $a\hat{\imath} + \hat{\jmath} + \hat{k}, \hat{\imath} + b\hat{\jmath} + \hat{k}$ and $\hat{\imath} + \hat{\jmath} + c\hat{k}$ $(a \neq b \neq c \neq 1)$ are coplanar, then find $\dfrac{1}{1-a} + \dfrac{1}{1-b} + \dfrac{1}{1-c}$.

(A) 0 
(B) 1 
(C) −1 
(D) 2

Solution: (B)


119. If matrix $A = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \dfrac{1}{k} adj(A)$, then $k$ is

(A) 7 
(B) −7 
(C) 15 
(D) −11

Solution: (C)


120. The angle between a pair of tangents drawn from a point T to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2 \alpha + 13\cos^2 \alpha = 0$ is $2\alpha$. The equation of the locus of the point T is

(A) $x^2 + y^2 + 4x - 6y + 4 = 0$

(B) $x^2 + y^2 + 4x - 6y - 9 = 0$

(C) $x^2 + y^2 + 4x - 6y - 4 = 0$

(D) $x^2 + y^2 + 4x - 6y + 9 = 0$

Solution: (D)